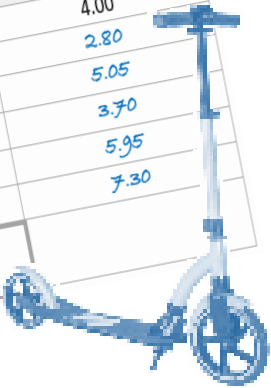# Describing relationships generally with variables and expressions

Date	Driving time (min)	Costs for actual driving time (€)	Total price as expression (€)	Total price calculated (€)
May, 16	20	20 × 0.15	20 × 0.15 + 1	4.00
May, 19	12	12 × 0.15	12 × 0.15 + 1	2.80
May, 24	27	27 × 0.15	27 × 0.15 + 1	5.05
May, 25	18	18 × 0.15	18 × 0.15 + 1	3.70
May, 29	33	33 × 0.15	33 × 0.15 + 1	5.95
June, 2	42	42 × 0.15	42 × 0.15 + 1	7.30
For every possible driving time:	X	X × 0.15	X × 0.15 + 1	

Related digital media The material includes four instructional videos; their links included via QR code at the relevant place. Tasks labeled T1, T2 enhance conceptual understanding using spreadsheets, but can be skipped if necessary.

Image rights All images are created by the authors or are credited.

A   Describing changing images in general terms

1   More and more tables, more and more chairs

a) The Özdemirs are a very hospitable family. No matter how many friends and relatives show up spontaneously, they always add tables and chairs. This goes on and on in the large garden.

1 table, 6 chairs   2 tables, 10 chairs   3 tables, how many chairs?
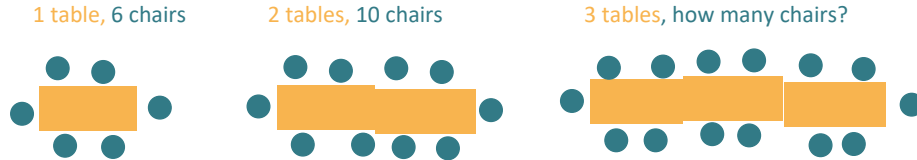
- How many chairs do you need for 3 tables, for 4 tables?
- How many chairs do you need for 8 tables? How can you find out without drawing?
- How many chairs do you need for 42 tables? How can you find out without drawing?

Number of tables	1	2	3	4	5	6	7	8	...	42
Number of chairs	6									

b) What does Merve mean by her claim?
How many chairs would you need for 17 tables?   Merve

> 2 at the ends and
> per table a set of 4 chairs.

c) Write down a calculation for how to find the number of chairs, if you know the number of tables.

2   Another sitting arrangement

a) Svetlana describes another arrangement of the tables at her parties.

> Each table seats 8 people, but the first table seats 2 less.

- Draw the arrangement.

1 table, how many chairs?   2 tables, how many chairs?   3 tables, how many chairs?

- For small parties, there are only three tables. How many chairs are needed?
- For bigger parties there are 16, 17 or sometimes even 22 tables.
  How many chairs are then needed?

| Number of tables	1					16	17	22	...	42
Number of chairs	6									

b) Write down a calculation for how to find the number of chairs, if you know the number of tables. Such a calculation is called *expression*.

c) How many chairs do you need for every possible number of tables?
How can you describe this with an expression?

3   Describing calculations generally

a) In the table, Pia wrote down her calculations for Task **1**.
Why does use "× 4"?
Can you add more expressions?

1 table, 6 chairs   2 tables, 10 chairs   3 tables, how many chairs?
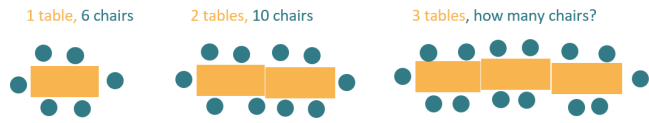
How many tables?	1	2	3	...	15	16	17	...	42	x
How many chairs?	2 + 4 = 6	2 + 2×4 = 10	2 + 3×4 = 14		2 + 15×4				2 + 42×4	

b) What does Pia mean by *x* in the last column? What could be the expression for the variable *x*?

c) For Task **2** Pia has also started writing expressions in the table.

> Each table seats 8 people, but the first table seats 2 less.

- Where does she use "-2"? Why "×4"? Show it in your drawing.

How many tables?	1	2	3		15	16	17	..	42	x
How many chairs?	8 – 2 = 6	2×8 -2 = 14	3×8 -2							

d)
- Fill in the table.
- Explain again, what does "every possible number of tables" mean?
- What could be the expression for "every possible number of tables"?

4   Understanding expressions with *x*

a) Pia wrote down an expression using the variable *x* again.
- Fill in the table by writing sample expressions with numbers instead of x.
  Therefore, *x* is called a *changeable number*.
- Draw the seating arrangement that matches this expression.

Number of tables	1	2	3	4	...	15	16	...	42	x
Number of chairs										2 × x + 4

b) Now, try explaining the expression 2 × *x* + 4:
- In the expression, *x* is multiplied by 2. Where can you find the 2 in the table?
- What does the *x* mean in the expression? Could you express it in another way?
- What can you calculate with the expression? Explain by using the table.

5 Series of matches

a)
- How many matches are needed for 3, 4, … , 42, … x triangles?
- Note the expressions with numbers first.

How many triangles?	1	2	3	4	…	15	16	42	x
How many matches?									

b)
- How many matches are needed for squares with 3, 4, …, 42 matches per side? Write down expressions with numbers.
- And how many for the xth square? Attention, sometimes you need x more than once in an expression

Side length of the square	1	2	3	4	…	15	16	42	x
How many matches?									

c)
- Why does the expression in **b)** differ from the ones before?
- Write down another expression for the squares in **b)**. To do so, structure them differently.

d) Lay or sketch two series of matches yourself for the expressions $2x^2$ und $4\times(x+1)$.

6 Towers of cubes

a) The first wall of cubes has 7 cubes.
How many cubes are needed for the 3rd, 4th , … 127th, … every possbile wall?
Write an expression using x that you can use to determine the number of cubes in the xth wall.

Wall number … ?	1	2	3	4	…	15	16	…	42	x
How many cubes?										

b) How many sides are exposed in the 1st, 2nd, 3rd, …, 42th, …. x^{th} figure?
Write down a series of numbers and write an expression for the xth figure .

Wall number … ?	1	2	3	4	…	15	16	…	42	x
How many sides ?										

B   Describing situations generally using changeable numbers

7   Describing the costs of riding an e-scooter

Lately Till has been renting e-scooters every now and then.
The following table shows how long he drove and what he paid.
The offer that Till uses is on the right.

a) • Fill in the missing values in Till's table.
You can choose **any** driving time for June 2nd.
• Write down headings for the columns.
• Complete the last row.

> **Service price**
> E-Scooter: 0.15 € per minute and
>   1 € for unlocking
> Standard tariff (0.15€/min) is charged
> when you unlock the scooter
> through the app.

Date	Driving time (min)			
May 16th	20	20 × 0.15	20 × 0.15 + 1	4.00
May 19th	12	12 × 0.15	12 × 0.15 + 1	
May 24th	27	× 0.15	+ 1	
May 25th	18			
May 29th		33 × 0.15		
June 2nd				
Every possible driving time:				

b) Explain using your own words:
• What does the expression in the **highlighted cell** of the last row stand for?
What can you calculate with it?
• What does "every possible driving time" mean to you?
• Why is the expression in the **highlighted cell** correct for every possible driving time?
• Why does Till have to multiply in the expression 20 × 0.15 on May 16th ?

The expression stands for:

"Every possible driving time" means:

This is why my expression is correct for "every possible driving time":

This is why Till has to multiply in 20 × 0.15:

8 Costs of riding an e-scooter – Describing changing numbers with variables

Preparing for the video:

a) Compare your results from Task **7**. Did you have difficulties with anything?
Do you have a question? Write down your questions here:

-
-
-

Working with the video:

Video 1:

b) Check your answers to **Task 7** against the video.

Complete the following sentences. Choose one of the options in **bold**.

https://educational-media.de/mum-video/interaktiv/watch/17

"Costs for actual driving time (€)" / "× minute price (€)" is a better heading for the 3rd column than "Costs for actual driving time (€)" / "× minute price (€)", because...

"A variable is a specific number that you can choose" **is correct / is not correct**, because...

"The expression $x × 0.15 + 1$ describes the variable driving time"
is correct / is not correct, because...

After the video:

c) Are your responses to **Task 7** correct? Correct, if necessary ...
- ...the entries in the table (headings, last row),
- .. your explanations in **7b)**.

d) Have your questions from **8a)** regarding Task **7** been answered?
Write down the answers here:

Answers to our questions regarding Task 7:

9   Costs of riding an e-scooter – Understanding general expressions

a) Pascal and Miriam made some mistakes when filling in the table.
Explain to them what went wrong.

> **Service price**
> E-Scooter: 0.15 € per minute and
>   1 € for unlocking

Pascal
> Driving time + minute price, because 0.15 are new in the column.

> Pascal, this is wrong because …

Need help with the answer for Pascal?
Watch the video again from **minute 1:15**

Miriam
> I always write down which quantity is new in the heading, so "+ unlock costs".

> Miriam, that is not completely wrong. But it is better if …
>
> Because…

Date	Driving time	Driving time + price per minute (€)	+ Costs for unlocking (€)	Total price calculated (€)
May 16th	20	20 × 0.15	20 × 0.15 + 1	4.00
May 19th	12	12 × 0.15	12 × 0.15 + 1	2.80
May 24th	27	27 × 0.15	27 × 0.15 + 1	5.05
May 25th	18	18 × 0.15	18 × 0.15 + 1	3.70
May 29th	33	33 × 0.15	33 × 0.15 + 1	5.95
For every possible driving time:	100	100 × 0.15	100 × 0.15 + 1	16.00

Pascal
> Every possible number means that I can choose a specific number. I simply choose 100.

> Pascal, this is wrong because …

Need help with the answer for Pascal?
Watch the video again from **minute 4:35**

b) Answer the questions posed by Farid and Yasmin:

Yasmin
> The general total costs x × 0.15 + 1 are with *x*. With this *x*, it is not possible to calculate something. What is the point of writing this down?

> Answer to Yasmin:

Need help with the answer for Yasmin?
Watch the video again from **minute 5:10**

Farid
> Why is the expression *x* × 0.15 + 1 **always** correct?
> We only tried 5 driving times.
> With a driving time of 1042 minutes, can't it be completely different?

> Answer to Farid:

T1   **Describing situations generally using spreadsheet**

In Till's spreadsheet, the same is calculated over and over again. Spreads can help with this.

> **Spreadsheets** are in a computer program. You can use them to calculate within tables automatically.

a) Get an overview of working with spreadsheets in the video.

b) Transfer Till's table to the file online.
The pictures will help you.

Writing a formula referencing cell B2:

	A	B	C	D
1	Day	Driving time in min	Costs for actual driving time	Total costs (including unlocking)
2	May 16th	20	=B2*0,15	
3	May 19th	12		

Transferring a formula in the bottom right corner to another row:

	A	B	C	D
1	Day	Driving time in min	Costs for actual driving time	Total costs (including unlocking)
2	May 16th	20	3.00 €	4.00 €
3	May 19th	12	1.80 €	
4	May 24th	17	2.55 €	

c) Note down formulas that will help Till calculate using the spreadsheet:
- Formulas to calcualte the cost for the actual driving time on May 19th .

- Formulas to calculate the total cost for his ride on May 24th .

d) Compute using the spreadsheet:
What would Till have to pay for 123.456.789 min driving time?

e) Nathan puts this expression into cell C2:
= B*0,15

	A	B	C	D
1	Day	Driving time in min	Costs for actual driving time	Total costs (including unlocking)
2	May 16th	20	=B*0.15	
3	May 19th	12		

But the spreadsheet can not work with this input.

	A	B	C	D
1	Day	Driving time in min	Costs for actual driving time	Total costs (including unlocking)
2	May 16th	20	#NAME?	
3	May 19th	12		

Explain:
- What is wrong with the input = B*0,15?

- What was Nathan presumably trying to express with his input = B*0,15 ?
Why is that input actually a *very good idea*?

10   **How much is a taxi ride? – Building complex expressions step by step**

Mary: There are many different costs riding taxis.

Till: And some costs are almost always different.

Pia: Can the total costs still be described generally somehow?

Taxi offer
- Basic fee   2.00 €
- Price per kilometer   1.75 € / km
- Price per minute spent waiting   0.85 € / min
  (e.g. headlights, traffic jam, …)
- Tip   optional

a) Discuss: Which quantities are the same for every ride? Which are usually different? Which changing quantities does the final price depend on?

b) Mary is constructing the expression for the costs of the taxi ride for example numbers first and then puts the expression together step by step. Explain her approach and fill in the gaps.

Mary: I'll start with an example.

1. Writing down changing quantities and choosing example values for the quantities:

Variable value	Specific example
• Number of kilometers	e.g., 5 km
•	
•	
•	

2. Building the expression step by step with helpful questions in between:

	Question	Expression	Add or multiply? Why?
(1)	How much do I pay for the driven kilometers?	5 × 1.75 ↓	Times, because I'm riding for 5 kilometers. For every kilometer I have to pay 1.75€, so 5 **times** the kilometer price, which is 5 **times** 1.75€.
(2)	How much do I pay for waiting time?	↓	
(3)	How much do I tip and how much basic fee do I pay?	↓	
(4)		5 × 1,75 + + 2.00	

c) What does Pia mean? Explain. How can all possible taxi rides be described?

Mary's expression in step 4 doesn't yet describe all possible taxi rides!

My idea for describing all possible taxi rides:

11 How much is a taxi ride? – Explaining why general expression are always correct

a) Till is trying to find the general expression for the Taxi offer in Task **10** by using a table.
- Write headings for each column.
- Complete Till's table.

Till

> A table helps me keep the overview. I need more examples. Then I can see which quantities change and which stay the same.

Changing quantities → → Complicated parts of the expression ↓ ↓

(in km)	(in min)	(in €)				Total price as an expression (in €)
20	3	5.00	2.00	20 × 1.75	3 × 0.85	
1		2.50	2.00		0 × 0.85	
	2	0.50		13 ×		
4	7	3.00				
For every possible taxi ride:						

b) Explain for the last row in Till's table: Why is the total price *variable*?
How can see this in the expression for the total price?

c) What problem does Pia have? Explain her question in your own words.

> Why is the expression *x* × 1.75 for the cost of the driven kilometers always correct?
> Till has only looked at 4 examples!
> What about all the other numbers of kilometers?
> Is the expression *x* × 1.75 even correct for them? Why?

Pia

d) How would you respond to Pia?

Dear Pia,
the expression also fits for other numbers of kilometers because …
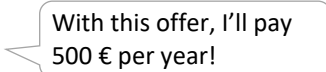
Helpful phrases for your answer:
- "*For every* kilometer you have to pay …"
- "The number of kilometers tells you how many times you have to pay the kilometer price."
- "The variable *x* stands for…"
- "No matter which number of kilometers you choose, …"
- "So you have to pay the kilometer price … times. That is … times 1.75."

e) Explain also why the expression *y* × 0.85 for the waiting costs is *always* correct.

12   How much costs the e-scooter annually? – Building complex terms independently

Yasmin wants to know how much an
e-scooter costs annually.

Another E-Scooter Offer
E-scooter: 0.10 € per minute and
             1.50 € for unlocking

Yasmin

> With this offer, I'll pay
> 500 € per year!
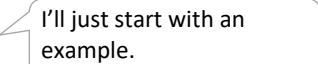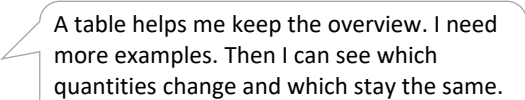
a) Vote whether Yasmin is right or not.

b) Build an expression for the e-scooter costs for a whole year.

Mary

> I'll just start with an
> example.

Hint: Build the expression with example numbers first,
like Mary in **Task 10b**, und generalize it afterwards
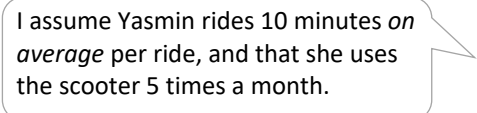with a table for many examples like Till
in **Task 11**.

Till

> A table helps me keep the overview. I need
> more examples. Then I can see which
> quantities change and which stay the same.

Steps that help with building general expressions

Building an example expression like Mary in Task 10b
- note changing quantities values and
  choose specific values
- build the expression step by step
  using helpful questions in between
  (Remember explaining why you use + or ×.)

> I assume Yasmin rides 10 minutes *on
> average* per ride, and that she uses
> the scooter 5 times a month.

- Finally, note down an expression for *every possible number* – for this, Till's approach might
  help:

Building a table like Till in Task 11
- build a table: each changing quantities value gets a column, complicated part of the
  expression too
- fill in different examples into your table
- in the last row, describe everything for *every possible number*

Final check:
- Is your general expression (in the last row of the table) actually *always* correct? Why?

T2 How much is the e-scooter annually? – General calculations using spreadsheet

a) Transfer the table from **12b** into a spreadsheet program.
The pictures remind you how the program can do calculations for you:

Writing a formula referencing cell B2:

	A	B	C	D
1	Day	Driving time in min	Costs for actual driving time	Total costs (including unlocking)
2	May 16th	20	=B2*0,15	
3	May 19th	12		

Transferring a formula in the bottom right corner to another row:

	A	B	C	D
1	Day	Driving time in min	Costs for actual driving time	Total costs (including unlocking)
2	May 16th	20	3.00 €	4.00 €
3	May 19th	12	1.80 €	
4	May 24th	17	2.55 €	

b) Investigate the following problem using the spreadsheet program:
On average, Yasmin drives for 5 min (10, 15, 20, 25 min) after unlocking.

- How many rides a month does it take for Yasmin to pay more than 500€ in a whole year?
- You can write your solutions down in the table.

Solution Drives in a month (amount)	Average driving time after unlocking (in min)	Total costs in a whole year (above 500 €?)
	5	
	10	
	15	
	20	
	25	

c) Yasmin wants to pay **less** than 500€ in a year and her rides are on average 20 minutes long.
How many rides can Yassmin afford each month?
Phrase your answer in 2 different ways. (The hint below may help.)

Your thoughts:

Hint: The ideas might be expressed like this:
- She can ride 4 times or more/less
- She has to stay above/below 4 rides
- She can only ride more/less than 4 times

d) Investigate the following problem using a spreadsheet:
Yasmin wants to pay less than 500€ per year and wants to ride about 15 times a month.

How long can her monthly rides be on average?

13 Filling knowledge store of Unit B

Complete knowledge store of Unit **B** by

(1) filling in the table,

(2) explaining the expression $1.5 + x \times 0.8$,

(3) explaining "every possible number",

(4) choosing what variables can be used for.

Knowledge Store B: Expressions with variables for changing numbers

If some numbers can change in calculations, the calculations can be written down using general expressions. In tables you can see *which numbers change* and *which remain the same*.

Example: Amir is training for a marathon. During training, he warms up first. Then, he runs a couple of big or small laps in the park.

Warmup	1.5 km
Big lap	0.8 km
Small lap	0.4 km

Changable numbers   **Complicated parts of the expression**

					Total distance in one training (in km)
		1.5			
		1.5			
For every possible training course:					
x	*y*		*x* × 0,8		

How to explain what an expression term including variables means:

Explain 3 questions:

1. What do the variables stand for?
2. Why did you choose these operations?
3. What exactly does the expression describe?

Example: 1.5 + *x* × 0,8

The variable *x* stands for …

For *x* × 0.8 I am using ×, because …

For 1.5 + *x* × 0.8 I am using + , because …

The expression 1.5 + *x* × 0.8 describes …

What "*x* stands for every possible number" means:

If *x* stands for a changeable number, you say *x* represents *every* possible number.

How to explain what "every possible number" means:

This is why variables as changeable numbers are useful:

(cross out what does not fit)

With variables, you can describe relationships such as training courses, transportation fees or total costs in general ways.

With variables, you don't need to calculate anything at all.

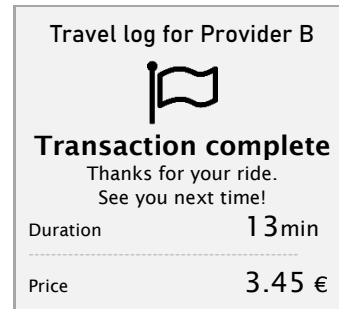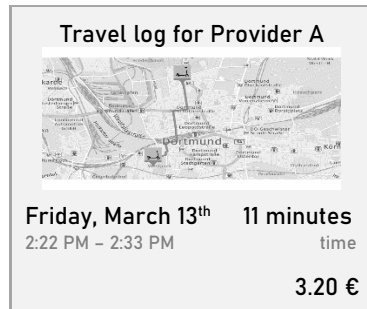With variables, you can show what numbers change in an expression.

Your explanation:

C   Determining unknown quantities

14   Setting up equations with unknown quantities and explaining them

a) Maya has used many providers of e-scooters. She is comparing her travel logs.
Which provider should she keep on using? Discuss your first ideas.

Travel log for Provider A	Travel log for Provider B
Friday, March 13th 11 minutes	Transaction complete
2:22 PM – 2:33 PM time	Thanks for your ride.
	See you next time!
3.20 €	Duration 13 min
	Price 3.45 €

b) Now take a closer look: Maya remembers that Provider A has an unlocking fee of 1€.
Can you find out what it costs to ride one minute after unlocking? Try examples.

c) Maya wants to know the driving fee per minute (**minute price**) of Provider A. To find out, she
writes down an equation: $m \times 11 + 1.00 = 3.20$. Explain what Maya's equation means:

1. The expression / number on the right side of the = stands for …

2. The m in Maya's equation stands for …

3. The expression on the left of the = stands for …

4. You put an equal sign "=" between the expressions to express that …

d) Provider B charges 0.20€ for unlocking. Write down a matching equation and explain what
it means. Use the beginnings from **c)**.

Equation:

1.

2.

3.

4.

15 Finding unknown values

Till: From this equation, we somehow have to find the unknown number.

Amir: I am going backwards in the arrow diagram

Maya: I use equations. Is this also possible?

a) Amir sketches how he builds the expression in a drawing and then tries to go backwards.
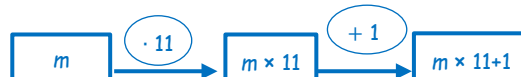- Watch the video to see how he thinks.
- Finish his calculation.

Video 3:

https://sima.dzlm.de/filme/sima-8-002-erklaervideo-3-rueckwaertsrechnen-unbekannte

Amir's arrow diagram:

Building the expression:

$$m \xrightarrow{\cdot 11} m \times 11 \xrightarrow{+1} m \times 11+1$$

Calculating backwards:

$$\boxed{} \xleftarrow{} \boxed{} \xleftarrow{-1} 3{,}20$$

b) Maya tries to find simpler equations that express the same as Amir's arrow diagram.
- How did Maya get the simpler equation $m \times 11 = 2.20$?
- How can she be sure that this equation $m \times 11 = 2{,}20$ is describing the same unknown value as $m \times 11 + 1.00 = 3.20$? Argue using the e-scooter offer.

Maya's simpler equations:

$$m \times 11 + 1{,}00 = 3.20$$

$$m \times 11 = 2.20$$

$$m =$$

c) Explain: How do Maya's and Amir's equations fit together?
Where can you find Maya's equations, e.g. $m \times 11 = 2.20$, in Amir's arrow diagram?

d) Answer Till's question.

Till: Why do you have to divide by 11 in the last step?

Maja: Let's think about what we distribute equally between the 11 ....

e) Calculate like Maya and Amir for **Provider B** (unlocking fee 0.20€). Draw an arrow diagram.

Maja's simpler equations:

$$= 3.45$$

$$=$$

$$m =$$

Amir's arrow diagram and calculating backwards:

16   Comparing two offers

Maja: Now I know what each provider costs. But I still don't know which is the better offer.

Till: What do we know?

a) Collect the information on the offers of Provider A and B from **Task 14** and **15**. Can you find expressions to describe the final price for every possible number of minutes driven?

Offer provider A:	Offer provider B:
Unlocking fee:	Unlocking fee:
Price per minute:	Price per minute:
Expression for final price for every possible number of minutes:	Expression for final price for every possible number of minutes:

b) Calculate in your workbook for providers A and B:
- How much does Maja pay for 10 minutes (20 minutes) of driving?
- How long can Maja ride with 3.20€ (4.20€, 5.00€)?
  You can use an arrow diagram or simpler equations.

Results for Provider A	Results for Provider B

c) Discuss again and then write it down: Which offer is better under which conditions?

17   Filling the knowledge store of Unit C

Complete the knowledge Storage of **Unit C** by ...

a) ... explaining the meaning of the equation $(1{,}75 \times x + 3.90) \times 2 = 49.80$;

b) ... solving the equation $(1.75 \times x + 3.90) \times 2 = 49.80$ using arrow diagrams or simpler equations;

c) ... explaining what variables standing for unknown numbers can be used for.

Knowledge store C: Equations with variables for unknown numbers

Sometimes you're looking for unknown numbers that meet certain criteria. This search can be expressed using equations.

> **Example:** Someone wants to go the same route back and forth by taxi. The following prices are charged:
> Basic fee   3.90 €
> Price per kilometre   1.75 €

How to explain what an equation with variables means in 4 steps:

Example: $(1.75 \times x + 3.90) \times 2 = 49.80$

1. The expression/number on the right side of the = stands for …

2. The x stands for….

3. The expression on the left side of the = stands for …

    You use the operation … because …

4. You put an equal sign = between two expression to express that ….

How to find unknown numbers by calculating backwards

Arrow diagram:

1. Translate the task into an arrow diagram:

2. Calculate backwards:

3. Write down the result and check it.

Looking for simpler equations

Find x so that
$$(1.75 \times x + 3.90) \times 2 = 49.80$$
$$=$$
$$=$$
$$x =$$

The unknown number is:

Check:

This is why variables as unknown numbers are useful
(cross out what doesn't fit)

The variable always represents all numbers.

The number that I don't know yet, I can write as the unknown x.

Using variables, you can put questions into equations.
For example: $(1.75 \times x + 3.90) \times 2 = 49.80$ is expressing a question.

Your explanation:

D   Distinguishing uses of variables

18   Comparing varying or unknown driving times

E-scooter: 0.15 € per minute and 1 € for unlocking

Till: In my expression $0.15 \times x + 1$ the x is really variable because it represents a changeable driving time. And the expression is variable too, so I write $y = 0.15 \times x + 1$.

Maya: Strange, in the equation $0.15 \times x + 1 = 3.10$, x is not variable anymore but fixed.

a) Explain in your own words what Till and Maya are trying to say.   *Video 4:*

b) Use the video to check your explanation.

c) Fill out the overview in the **Knowledge Storage of Unit D**.
To do so, cut out the texts, match them to the correct use of the variable, and complete the texts if necessary.

d) Josef: I don't get the difference between variable as changing quantities and as unknowns! For $y = 0.15 \times x + 1$ I don't know the x either. So why is it changeable and not unknown?

Help Josef to understand the difference.
- Explain using the table: Why is the x in $y = 0.15 \times x + 1$ changeable?
- What would need to change so that x describes an unknown number? Explain why x is then an unknown.

Date	Driving time (min)	Costs actual driving time (€)	Total price as expression (€)
May, 16	20	20 × 0.15	20 × 0.15 + 1
May, 19	12	12 × 0.15	12 × 0.15 + 1
May, 24	27	27 × 0.15	27 × 0.15 + 1
May, 25	18	18 × 0.15	18 × 0.15 + 1
May, 29	33	33 × 0.15	33 × 0.15 + 1
June, 2	100	100 × 0.15	100 × 0.15 + 1
For every arbitrary driving time	x	x × 0.15	x × 0.15 + 1

Dear Josef,

$x \times 015 + 1 \overset{?}{=} 3.10$

The question mark fits the variable as

_____ ,

because …

$1 \times 0{,}15 + 1$	$21 \times 0{,}15 + 1$	$41 \times 0{,}15 + 1$	$61 \times 0{,}15 + 1$	$81 \times 0{,}15 + 1$
$2 \times 0{,}15 + 1$	$22 \times 0{,}15 + 1$	$42 \times 0{,}15 + 1$	$62 \times 0{,}15 + 1$	$82 \times 0{,}15 + 1$
$3 \times 0{,}15 + 1$	$23 \times 0{,}15 + 1$	$43 \times 0{,}15 + 1$	$63 \times 0{,}15 + 1$	$83 \times 0{,}15 + 1$
$4 \times 0{,}15 + 1$	$24 \times 0{,}15 + 1$	$44 \times 0{,}15 + 1$	$64 \times 0{,}15 + 1$	$84 \times 0{,}15 + 1$
$5 \times 0{,}15 + 1$	$25 \times 0{,}15 + 1$	$45 \times 0{,}15 + 1$	$65 \times 0{,}15 + 1$	$85 \times 0{,}15 + 1$
$6 \times 0{,}15 + 1$	$26 \times 0{,}15 + 1$	$46 \times 0{,}15 + 1$	$66 \times 0{,}15 + 1$	$86 \times 0{,}15 + 1$
$7 \times 0{,}15 + 1$	$27 \times 0{,}15 + 1$	$47 \times 0{,}15 + 1$	$67 \times 0{,}15 + 1$	$87 \times 0{,}15 + 1$
$8 \times 0{,}15 + 1$	$28 \times 0{,}15 + 1$	$48 \times 0{,}15 + 1$	$68 \times 0{,}15 + 1$	$88 \times 0{,}15 + 1$
$9 \times 0{,}15 + 1$	$29 \times 0{,}15 + 1$	$49 \times 0{,}15 + 1$	$69 \times 0{,}15 + 1$	$89 \times 0{,}15 + 1$
$10 \times 0{,}15 + 1$	$30 \times 0{,}15 + 1$	$50 \times 0{,}15 + 1$	$70 \times 0{,}15 + 1$	$90 \times 0{,}15 + 1$
$11 \times 0{,}15 + 1$	$31 \times 0{,}15 + 1$	$51 \times 0{,}15 + 1$	$71 \times 0{,}15 + 1$	$91 \times 0{,}15 + 1$
$12 \times 0{,}15 + 1$	$32 \times 0{,}15 + 1$	$52 \times 0{,}15 + 1$	$72 \times 0{,}15 + 1$	$92 \times 0{,}15 + 1$
$13 \times 0{,}15 + 1$	$33 \times 0{,}15 + 1$	$53 \times 0{,}15 + 1$	$73 \times 0{,}15 + 1$	$93 \times 0{,}15 + 1$
$14 \times 0{,}15 + 1$	$34 \times 0{,}15 + 1$	$54 \times 0{,}15 + 1$	$74 \times 0{,}15 + 1$	$94 \times 0{,}15 + 1$
$15 \times 0{,}15 + 1$	$35 \times 0{,}15 + 1$	$55 \times 0{,}15 + 1$	$75 \times 0{,}15 + 1$	$95 \times 0{,}15 + 1$
$16 \times 0{,}15 + 1$	$36 \times 0{,}15 + 1$	$56 \times 0{,}15 + 1$	$76 \times 0{,}15 + 1$	$96 \times 0{,}15 + 1$
$17 \times 0{,}15 + 1$	$37 \times 0{,}15 + 1$	$57 \times 0{,}15 + 1$	$77 \times 0{,}15 + 1$	$97 \times 0{,}15 + 1$
$18 \times 0{,}15 + 1$	$38 \times 0{,}15 + 1$	$58 \times 0{,}15 + 1$	$78 \times 0{,}15 + 1$	$98 \times 0{,}15 + 1$
$19 \times 0{,}15 + 1$	$39 \times 0{,}15 + 1$	$59 \times 0{,}15 + 1$	$79 \times 0{,}15 + 1$	$99 \times 0{,}15 + 1$
$20 \times 0{,}15 + 1$	$40 \times 0{,}15 + 1$	$60 \times 0{,}15 + 1$	$80 \times 0{,}15 + 1$	$100 \times 0{,}15 + 1$

$x \times 0{,}15 + 1$

The image matches the variable as

_____ ,

because …

Situation: Taxi

$y = 4 + 1.5 \times x + 2.5$

Situation: Taxi

$4 + 1.5 \times x + 2.5 \overset{?}{=} 13$

You can plug in numbers into the variable, but no specific number is searched.

It can appear in expressions.

It's not about finding a hidden number but describing a relationship generally.

The number is represented by letters.

You can write calculations for all possible numbers with it.

Task for a situation: Lara's training

Lara runs a different amount of laps every day. She doesn't want to always think about how many kilometers she ran. That's why Lara wants to find an expression for the complete distance.

Question: What is the expression?

Answer:

Task for a situation: Lara's training

Today, Lara's training schedule tells her to run 4.7 kilometers in total. Now Lara wants to know …

Question:

Answer:

Knowledge store D: Variable as changeable quantity and as unknown

Variables are used differently:

Variable: x

Variable as changeable quantity   **Variable as unknown number**

Situation: Taxi
Prices
Basic fee:	4.00 €
Price per km:	1.65 €
Tip:	2.50 €

Situation: Lara's training

Laras training for a half marathon. She starts every training with running 1.5 km to warm up. Afterwards, she runs an arbitrary number of laps around the training court. One lap is 400m.

Task for a situation: Lara's training   *Task for a situation: Lara's training*

19 Changeable quantities or unknown numbers in different situations

a) Use the example to explain what Maya has observed:

> **E-scooter**: 0.10 € per minute and 1 € for unlocking

For every driving time, you can describe the total price as follows: $y = 0.10 \times x + 1$.

How long will I drive if I pay 2.30 €?

$0.10 \times x + 1 = 2.30$

$\qquad 0.10 \times x = 1.30$

$\qquad\qquad x = 13$

> Whether *x* describes all possible numbers or only an unknown number can change from one step to the next!   Maja

b) For every variable in the boxes, imagine
- a situation and an equation in which the variable is a *changeable quantity*.
- a situation and an equation in which the variable is an *unknown number*.

Explain for your story why it is helpful to use a variable.

n is an amount of students	*t* is a number of minutes
Idea: Entrance to a museum with teachers	Idea: International calls with basic fee