DZLM German Center for Mathematics Teacher Education

Using variables to describe every possible number

Text for the interactive video

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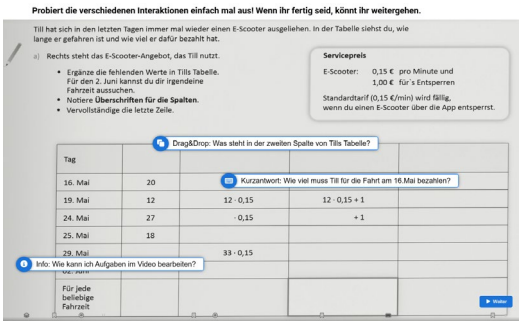
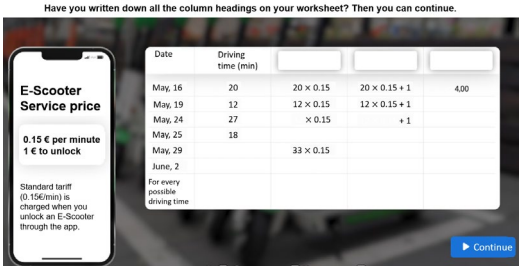
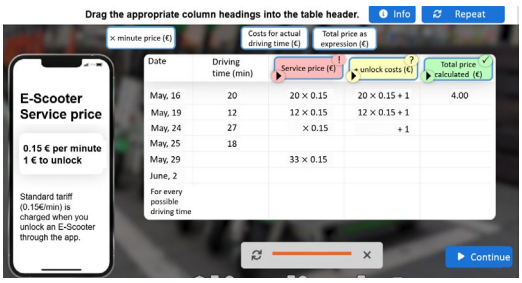
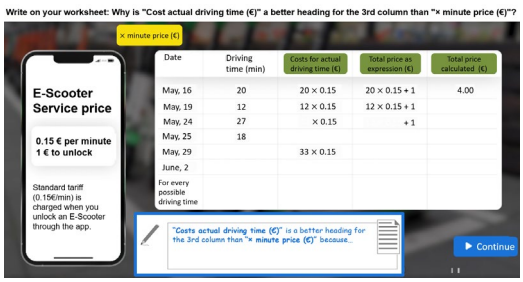
As part of the language-responsive teaching material developed by Stefan Korntreff & Susanne Prediger

Link to the instructional material and video: https:// sima.dzlm.de/um/8-002

	Animation and text of the video (most important meaning-related language in **bold**)	Comment for teachers
0:00	**Getting to know the interactive features**	
		The following video is designed to get you active. By clicking and dragging you can answer questions and get additional information. You will be given an example to try out, so later you will know how it works.	• The video offers drag-and-drop tasks, open-ended responses, and single-choice questions. Some of these interactive features can be tried at the beginning to help students later focus on the content.
0:23	**Making sure that relevant prior knowledge is activated** This video shows you how to use variables to describe **every possible number**. Let's go back to the example with the e-scooter rides.		• The video helps students to develop an understanding of variables as generalizers that can describe all possible numbers simultaneously. It also promotes an understanding of expressions as descriptions of general relationships.
		In the first two columns of Till's table, we can see on which days he used the e-scooter and for how long. For the remaining columns, we will take a closer look at the headings. It helps if you have entered all the column headings on your worksheet. Have you already done that?	• The first interactive feature ensures that all students have activated their prior understanding of what the expressions in the columns describe.
1:16	**Interactive features addressing the meaning of the expressions in the columns**		
		We will now suggest some headings for the last three columns. Which one do you think is the best? As you answer, explain to each other why you chose a particular heading.	• The interactive features addresses typical conceptions of students regarding the headings. Students often focus only on the "new" quantities in the columns ("+ unlocking costs"). The audio feedback helps to describe the expressions as a single quantity ("cost for the actual/pure driving time").

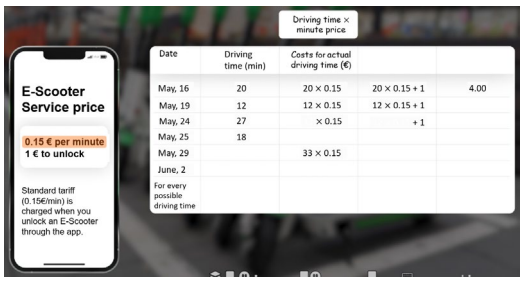


[*Explanation prompt at the end of the drag-and-drop task:*
Write on your worksheet: Why is "Costs for actual driving time (€)" a better heading for the 3rd column than "× minute price(€)"]

- This prompt is designed to help students think through the description of the expression as a single quantity rather than as a prompt to calculate.
- The accompanying worksheet (Task 8b) provides sentence starters for responses.

1:16 **Interpreting the expressions with a horizontal view on the table**

Let's take another look at the column headings together:
On May 16th, Till used the e-scooter for 20 minutes.
According to the offer, e-scooter use costs 15 cents per minute.

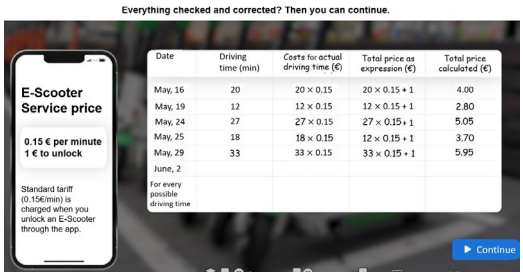


So, Till pays 20 times 15 cents for the time he drives, that means **20-times the price for one minute**. In the third column, we calculate the driving time times the price per minute. The **whole expression describes the costs for the actual driving time** without unlocking.

Unlocking the e-scooter costs an additional 1 Euro – no matter how long you drive afterwards. This euro is simply added to the costs for the actual driving time in the penultimate column. So, here is the sum of the actual driving time costs and the unlock costs.
This is the **expression for the total price of the ride**.
In the last column, we finally calculate the total price.
For the 20-minute ride on May 16th, its 4 Euro.

- When interpreting expressions, care is taken to first interpret all the quantities and operations of the expression separately ("20 times the price of one minute"). After that, the expression is interpreted as a single quantity ("cost for actual driving time"), this is a very important step in students' understanding of expressions.

2:19 **Competing the table for the "May rows" with a stop moment**



Now, we can complete the table. For the next four rows, it looks like this. Check the numbers in your table and correct them if necessary.

- The built-in pause allows students to compare (at their own pace) the table entries with their responses. This reduces the pressure during the later consolidation phase, so the focus is on the meaning of the expressions and on the variable as a generalizer.

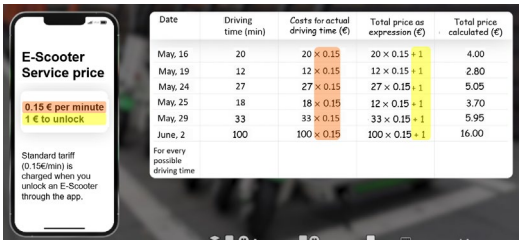
2:34 **Any driving time: Completing the second last row of the table**

In the second last row you could choose **any driving time**. Let's take 100 minutes – that is easy to calculate [*second last row is filled in step by step for 100 minutes, during which time the narrator is silent*].
Of course, you could **have chosen** a completely different number, since you were asked for **any driving time**.

- The distinction between "any driving time" for variables as placeholders and "every possible driving time" for variables as generalizers is crucial as it highlights the subtle differences between the concepts.

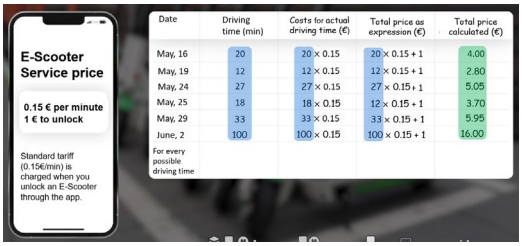
2:57 **Explicating the vertical structure of the table**

Let's review the previous entries:



You see that the 0.15 appears in each row. That's the price in euros, which is fixed for one minute of driving time. The 1 also appears in each row. This is the one euro for unlocking. And – just like the price per minute – it **doesn't depend on** how long you drive.

- Colouring the quantities in the columns allows students to better perceive the general structure of the expression: Which quantities always remain the same? Which can change? This prepares the student for describing the changing quantities in general terms, for example, by using a variable.

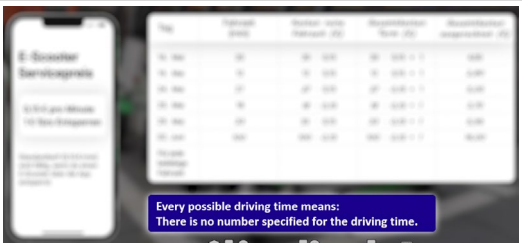


The driving time, on the other hand, **is different in each row** and, therefore, the costs for the actual driving time as well as the total price also **change**. Both ultimately **depend on** the driving time.
So, there are quantities that **remain the same in each row**: the price per minute and the unlock costs. And there are **quantities that change in each row**, such as the driving time and the total price

3:49 **The meaning of "every possible driving time"**

How does it look now for the last row?
Till's driving time can have very different values: For example, 20, 12, or 33 minutes as in the table, but also 1 or 1000 minutes.

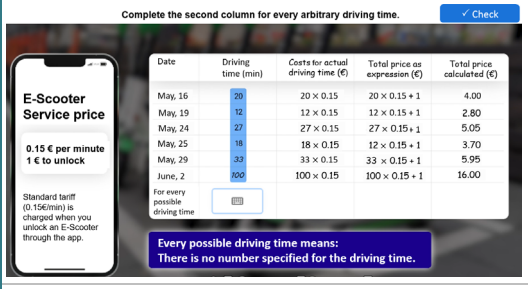
- Listing different values of the driving time and pointing along the 2nd column emphasizes the idea of changing quantities.



That's why we want to describe **not just any chosen driving time, but rather every possible driving time in general**. So we no longer specify how long Till drives. [Blue *mnemonic sentence is displayed in silence*]

- The language expressions "any chosen" and "every possible" ("no specific number is fixed") highlight the difference between placeholder and generalizer explicitly.

4:29 **Interactive features regarding the meaning of "every possible driving time"**



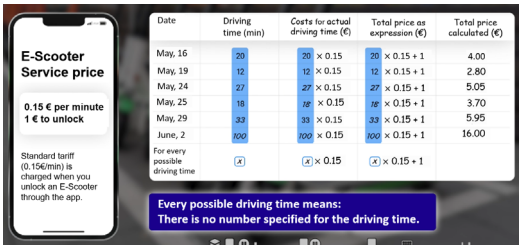
Now we can fill in the last row. First, complete the driving time column.

- This open-ended gap encourages students to think deeply about the mnemonic sentence. The audio feedback emphasizes the difference between any and every possible driving time.

4:29 **Explanation of the meaning of "every possible driving time"**

Since the driving time is not specified, we cannot write down a specific number. Instead we use a letter variable, for example x. This expresses that Till drives the e-scooter for x minutes on any given day. x stands for **all possible driving times that Till could ever drive**.

- "all possible numbers" is another language expression that emphasizes the meaning of the generalizer.



The remaining columns emerge quickly: If Till drives for x minutes, then he also has to pay x times 15 cents for the time he drives.
If we add the 1 Euro for unlocking, we get **the total price for every possible ride**.

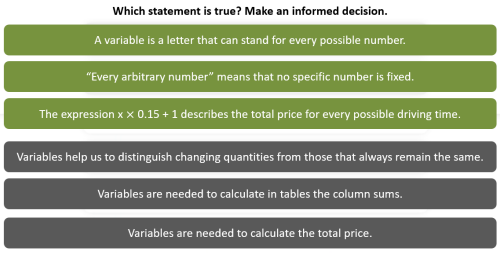
- Here, the expression is described as a general single quantity. During the consolidation phase, it is worth discussing the meaning of the condensed description "the total price for every possible ride " in detail with students, as this summarizes all the relevant information.

In the last column, we cannot calculate a specific amount anymore. After all, Till's **driving time is not specified**, so the total price is not specified either.
Instead, the total price is given by the expression "x times 0.15 plus 1" – **for every possible driving time x**.

So, with a single expression, we have grasped **the total price for all possible driving times**. And there is another benefit. The variable makes it clear that **the driving time changes**. Unlike the price per minute and the unlock cost, which **always remain the same**.

- Here, two reasons are provided why variables as generalizers are useful. This sense-making is essential for building an understanding and should be emphasized repeatedly in further experiences with generalization activities.

5:51 **Interactive feature „Summary"**



Now, check the meaning of important concepts that played a role in this video.
[*Explanation prompts at the end of the summary task:*
Write on your worksheet: Why is this response not completely correct:

- *"A variable is a specific number that you can choose"?*
- *"The expression x×0.15+1 describes the variable driving time"*]

- This interactive feature summarizes the meanings of the most important concepts from the video. It is worth discussing with the students in the consolidation phase why the wrong answers are not (completely) correct.
- Two explanation prompts encourage students to think through the correct answers as opposed to the (not quite) correct answers. The accompanying worksheet (Task 8b) provides sentence starters for students' responses.

5:59 **Conclusion**

In this video, you saw that you can use a variable to describe every possible number. Therefore, for example, we can write down the total price of all possible driving times that Till could ever drive.
Variables also help distinguish changeable quantities, such as the driving time, from quantities that remain the same in every calculation.