



Unpacking mathematical concepts in multiple languages

Examples of triangle, perimeter and even numbers

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Project affiliation

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Links and reference to other materials

Two films for teachers explain the mathematics education background of the material under sima.dzlm.de/um/multiple-languages

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Background of the teaching material for teachers

Dear mathematics teachers,

With this teaching material, we aim to give an idea of how you can use multiple languages in your mathematics classrooms. We can demonstrate our appreciation of language diversity by valuing the linguistic diversity the students bring and permitting the use of multiple languages in pair and group work. This can empower students to engage in more intense communication, which is important at a general education level. Encouraging communication in multiple languages supports students in building bridges from everyday life into mathematics and back.

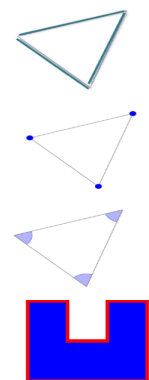
Nevertheless, beyond these pedagogical and communicative purposes, we can also exploit the power of multiple languages for *mathematics learning* itself, i.e., for epistemic purposes. Our approach is to identify mathematical potentials, establish and develop language comparisons of the literal meanings of terms and then initiate discussions about meanings through language comparisons.

We have produced two films for teachers (sima.dzlm.de/um/multiple-languages) that explain the mathematics education background of this material under sima.dzlm.de/um/multiple-languages. These films also contain scenes from mathematics classrooms showing the idea: When students break down terms for mathematical concepts from other languages into their literal meanings, then, different aspects of meaning can sometimes emerge. When this happens a more multifaceted view of the whole concept becomes available than if the concept is discussed in only one language. This can encourage students to explain the meanings of concepts in the first place, and to connect several meanings to each other. Consequently, the language comparisons provide great opportunities to initiate thinking and talking about mathematics by unpacking terms from multiple languages.

To do this, however, we first need to introduce students to the idea of translating not just by overall sense, but literally. This makes language reflection possible. Later, we can build on this by eliciting further technical terms and their literal translations from multilingual students. This positions these students as knowers and valuable contributors to classroom discussions.

The teaching material contains reflections on three mathematical concepts:

- The concept of **Triangle** is used to introduce the practice of language comparison in a conceptually simple case: **Tri-ángulo** (Spanish for “three-angle”), **Drei-eck** (German for “three-vertex”), **Tre-kant** (Danish for “three-side”); each refers to different components of the triangle. By discussing differences, students become aware that unpacking words can be mathematically interesting.
- **Area and perimeter** provide the second example, and focuses on conceptually challenging students to clearly distinguish between them. **Omtrek** (Netherlands for “around-track”) speaks nicely and is easy to distinguish from **Emvadón** (Greek “go into something”).
- **Odd and even** is the third example: The different terms for odd and even in multiple languages refer to two models of division: **gerade Zahl** (German for “levelled-out number”) uses the idea of partitive division, while **çift sayılar** (Turkish for “couple number”) refers to quotitive division.



Identify mathematical potentials in multiple language comparisons



Establish and practice language comparisons on literal meanings



Initiate discussions about meanings through language comparisons



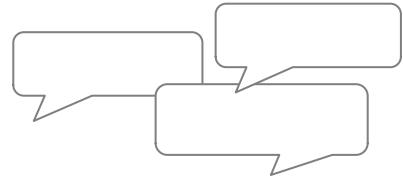
Levelled number		Pair number	
A number is even if it can be split into two equally large groups.		A number is even if it can be split into pairs, i.e., groups of 2.	
Then the plane is not uneven, but levelled out, that means even.		Then, nobody is sad, because it is single.	
gerade Zahl		partall (Paar-Zahl)	
even number (Ebene-Zahl)		çift sayılar (Pärchen-Zahl)	
		número par (Paar-Zahl)	

Even when the area/perimeter or odd/even are taught separately, we suggest to start with the triangle example to sensitize students for language reflections on literal meanings.

Suggestion for a possible lesson plan (planned for 135 min + 30 min homework)

(Lesson plan to be flexibly adapted according to the needs of each class)

Task Activity setting	Learning goal and steps to the learning goal	Time
1	Triangle – Dreieck – Trekant	20 min
	Introduce students to language comparison practices by unpacking literal meanings	
a) Plenary	• Brief question for unpacking first word: triangle	3 min
b) Plenary	• Explain expectation for task b) (not the solution)	
Pairs	• Briefly match terms Triangle – Dreieck – Trekant to three figures that emphasize different components (students intuitively search for meanings)	3 min
Plenary	• Justify the matches and explain how to unpack meanings: How have you decided what to compare? (students are introduced to unpacking literal meanings)	4 min
c) Individual	• Write down explanations (students consolidate what was discussed before)	3 min
d) Individual	• Individual search for other languages (use google translate)	4 min
Plenary	• Briefly exchange further examples (teachers monitor whether translations provided overall sense or were literal)	3 min
2	Perimeter and area: Distinguishing concepts by unpacking terms	25+20 min
a) Individual	• Remember what area and perimeter are by calculating an example	3 min
b) Individual	• Write down explanation for difference between perimeter and area	2 min
Plenary	• Students present two or three explanations	4 min
c) Plenary	• Unpack “Omtrek” (introduce practice of unpacking)	3 min
de) Pairs	• Unpack terms from multiple languages and explain the assumed meanings	5 min
Plenary	• Students present their explanations	8 min
<i>If you have only one 45-minute lesson, a cut can be here to continue with 2f) in the next lesson.</i>		
f) Plenary	• Briefly discuss German mnemonic bridge – prepositions in all the words	2 min
g) Plenary	• Explain expectation for task g) (not the solution)	1 min
Pairs	• Write questions for perimeter and area for different situations	10 min
Plenary	• Brief exchange for written questions (teacher monitors whether all students now distinguish area and perimeter correctly)	5 min
Input	• Teacher summarizes, e.g.: “Sometimes, terms in other languages are great to unpack the meaning of a mathematical concepts. Other languages might speak to us better than the official language of instruction. Make use of this in the future!”	2 min
3	Odd / even numbers – Part 1: Mobilize students own ideas on odd and even	25 min
ab) Individual	• Draw an image, what is an odd number? What could a “single number” mean?	8 min
Plenary	• Exchange ideas from a) and b)	8 min
c) Plenary	• If the students did not have a good idea for a diagram in a) where to describe “levelled out”, continue with 4a) in which the dot diagram is introduced	9 min
4	Odd / even numbers – Part 2: Understand division in two models by unpacking terms	20 min
ab) Pairs	• Draw and discuss diagrams for odd and even, include a levelled-out bar and an uneven bar, make sense of a “couple number”	10 min
Plenary	• Exchange ideas and explanations	10 min
5	Odd / even numbers – Part 3: Systematize terms for two models of division	25 min
a-c) Pairs	• Assign terms to two models of division: “levelled number” (partitive division in two equal groups) vs. “pair number “quotitive division into groups of two)	8 min
Plenary	• Exchange ideas	8 min
Individual	• Write down individual learning gain	5 min
Plenary	• Make a brief concluding meta-reflection on the approach of unpacking mathematical concepts in multiple languages.	2 min
6 Homework	Odd / even numbers – Part 4: Sum of two numbers	30 min



Dear students and teachers,

There is power in words, and languages sometimes have different powers!
Let's think about this today, using three examples.

We will unpack terms for mathematical concepts in multiple languages
and see that they can have slightly different meanings.
Sometimes other languages have terms that explain concepts particularly well.
So it's worth taking a closer look.
This allows you to think about the concepts from several perspectives.

Why not give it a try!

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1 How to speak about triangles?

- a) What does the term **triangle** mean literally? Do you think the name is suitable? Why or why not?
- b) In German, a triangle is called **Dreieck** (literally: three-vertices), and in Danish **trekant**. What could **tre**, **kant** mean, what do you think? And **triángulo** in Spanish? Which figure best visualizes the meaning of each of the terms? Connect each term to the figure that emphasizes the important part.

Trekant

Danish

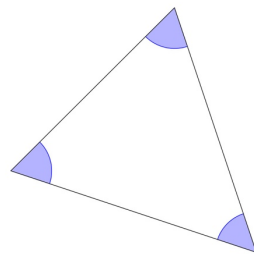


Figure A

Triángulo

Spanish

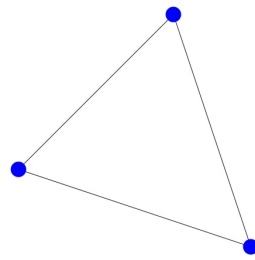


Figure B

Dreieck

German

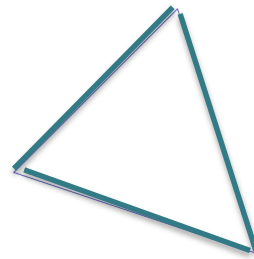
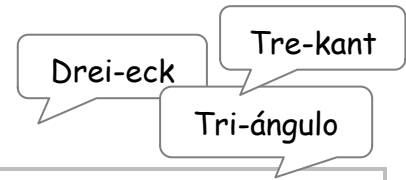


Figure C

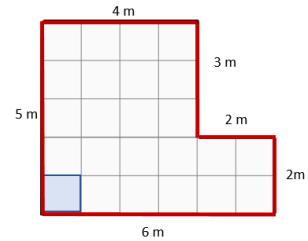
- c) Explain in writing what the three terms have to do with each other. Do they show the same figure? Why do you think that?



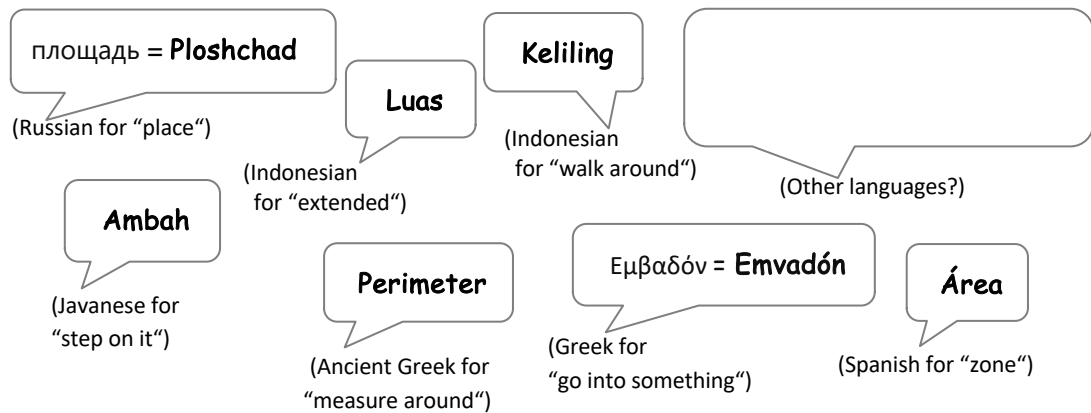

- d) Find out what the triangle is called in other languages. Try to translate these terms literally into your languages. Do you think the name is appropriate? Why do you think that?

2 Perimeter and area

- a) Determine the perimeter and the area of this figure.
- b) Some students mix up area and perimeter. How could you explain the difference?



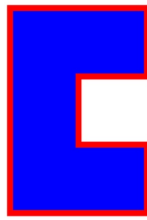
- c) In Dutch, they use the term **Omtrek**. Do you think this word means **area** or **perimeter**? Why do you think that?
- d) When you translate words from other languages literally, you can discover exciting things. Which terms stand for **perimeter**? Which for **area**? Do you know any other words for these terms from other languages?



- e) Choose a term from **d)** that you think stands for **perimeter**. Explain why you think this. Choose a word from **d)** that you think stands for **area**. Explain why you think this.



- f) A German textbook offers the following help for memorizing:
Umfang contains **Um** (“around”) and **Flächeninhalt** contains **In**.
 Can you find something similar insights in the terms in your language?



■ ■ fang ist dr ■ ■ her ■ ■,
 Flächen ■ ■ halt ist ■ ■ nen dr ■ ■.

- g) In some situations, we are interested in the perimeter, in other situations in the area, sometimes in both.
- Lisa has written a question for Situation A. Does she ask for the perimeter or the area?
 - For which situation can you ask a question about the perimeter?
 - For which situation can you ask about area?

Question for Situation A:
 Cold wind comes in through the window frames, so Auntie Emma buys insulating tape. How many meters of tape does she need?

Situation A:
 Window



Situation B:
 Sandpit



Situation C:
 Stadium



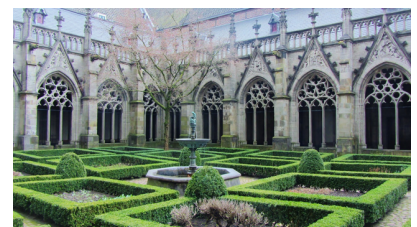
Situation D:
 Swimming pool



Situation E:
 Horse pasture



Situation F: Monastery courtyard and monastery tour



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3 Odd and even numbers

- a) What is an odd number?
- Explain in your words and draw an image to explain it.
 - How does the image show that the number is odd?



- b) **Odd** numbers, are called **أرقام فردية** in Arabic, which literally means “**Single number**”.
- Explain what could be meant with **single number**.

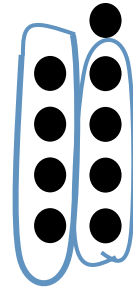


- c) In German, they use **gerade Zahl**, literally, this means “**levelled out number**”.
- Do you think this term is for odd or even numbers? Why do you think that?
 - Can you see the levelled out amounts in your image from **a)**?
 - The other German word is **ungerade**, what do you think that means?



4 Odd and even numbers – Part 2

- a) Kenan has drawn this image for the number 9.
- How does this image show that 9 is odd?
 - Draw similar images for the numbers 4 and 5.
 - How can you see that they are **gerade / ungerade**?
 - Can you use this to explain what **gerade / ungerade** and the English term **even** mean?



- b) In Turkish, even numbers are called **çift sayılar**, which literally means "**couple number**".
- Explain what **couples** have to do with even numbers.
 - Can you draw another dot image similar to Kenan's, for **couple numbers**?
 - What is different to Kenan's image in a)?

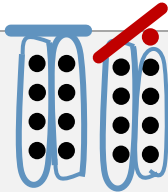
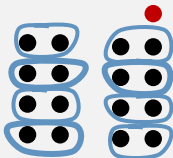


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Dancing pairs: https://de.freepik.com/vektoren-kostenlos/leute-tanzen_4012052.htm#fromView=keyword&page=3&position=50&uuiid=2b1ab5d5-178c-4a7b-bc14-cc06b75c3e6e

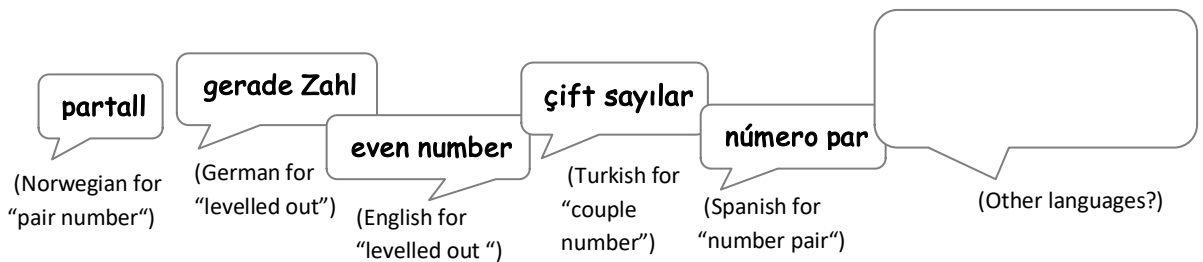
5 Odd and even numbers – Part 3

In mathematics, even numbers are defined as those which can be divided by 2 without any remainders. This technical definition is the same in all countries. But how we understand and visualize divisibility by 2, this differs in different languages:

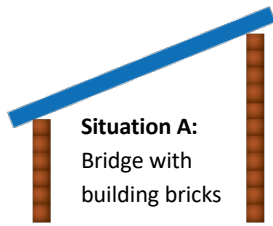
<p>Levelled number A number is even if it can be split into two equally large groups. <i>Then the bar is not uneven, but levelled out, that means it is even.</i></p> 	<p>Pair number A number is even if it can be split into pairs, i.e., groups of 2. <i>Then, there is no sad single unit.</i></p> 

- a) Explain again the difference between **levelled numbers** and **pair numbers**. Which description do you prefer?

- b) Fill in the terms from different languages into the right columns. Which goes better with the **levelled number**, which better with the **pair number**? Which further terms do you know or can you find in a dictionary?



c) Which situation matches better with the **levelled number** / the **pair number**, which one to both?

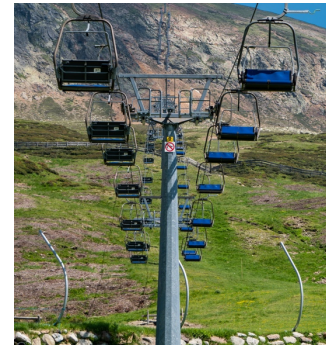


Situation A:
Bridge with building bricks

Situation C:
Chess competition



Situation D:
Cable car



Situation B:
Couples dance



d) What did you learn about even and odd numbers today?
What did you learn about how multiple languages and words can help you understand mathematics?

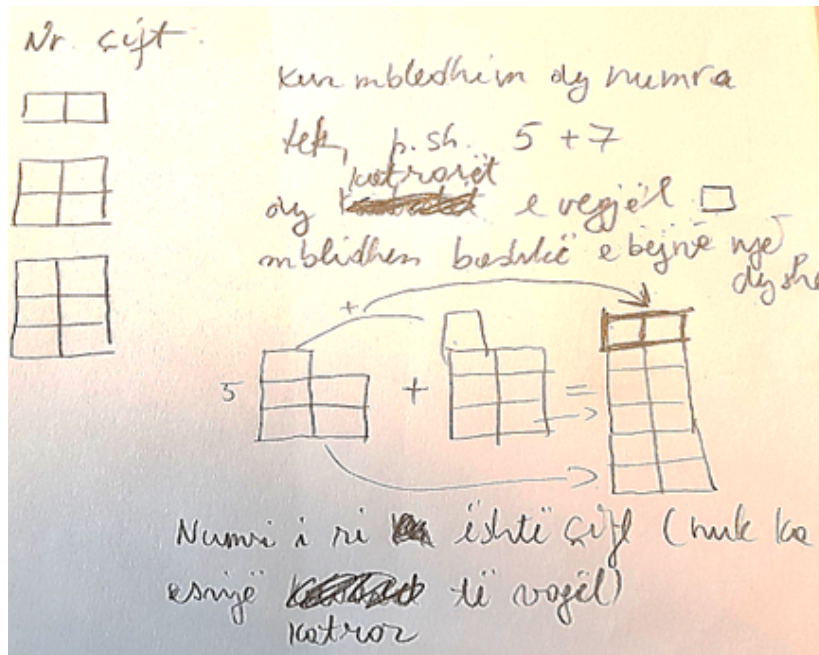


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6 Odd and even numbers – Part 4: Sum of odd number

- a) When you take the sum of two odd numbers: What will you get? Even or odd? Why?
- b) Below you can see the explanation of an Albanian-speaking student, Tarik.
- Can you explain what Tarik was drawing?
Which parts of the argument can you understand without knowing Albanian?
 - Tip: The Albanian word çift means pair, as in Turkish.

Tarik's argument that the sum of two odd numbers is...



- c) Tarik has argued for $5 + 7$. Can you generalize the argument for other odd numbers? And to any odd numbers, no matter which two numbers you choose?
- d) Based on Tarik's drawings and the Arabic idea of describing odd numbers as single numbers, write a text in one of your languages that explains why the sum of two odd numbers is always an even number, no matter which two odd numbers you choose.